Measuring Rate

Unit Rate is often a useful means for comparing ratios and their associated rates when measured in different units. The unit rate allows us to compare varying sizes of quantities by examining the number of units of one quantity per 1 unit of the second quantity. This value of the ratio is the unit rate.

Example: Janet walks a distance of $\frac{4}{5}$ mile in $\frac{1}{10}$ hour. What is the average speed Janet walks per hour?

\[
\begin{align*}
\text{miles} & \quad \frac{4}{5} \quad \frac{1}{10} \\
\text{hours} & \quad \frac{1}{10} \\
\text{Unit Rate} & \quad \frac{4}{5} \div \frac{1}{10} = \frac{4}{5} \cdot \frac{10}{1} = \frac{40}{5} = \frac{8}{1} \\
& = 8 \text{ miles per hour}
\end{align*}
\]

Identifying Proportional and Non-Proportional Relationships in Tables

One quantity is proportional to a second if there exists a constant (number) such that each measure in the first quantity multiplied by this constant gives the corresponding measure in the second quantity.

Steps to determine if two quantities in a table are proportional to each other:

- For each given measure of Quantity A and Quantity B, find the value of $\frac{B}{A}$.
- If the value of $\frac{B}{A}$ is the same for each pair of numbers, then the quantities are proportional to each other.

Example: The table below shows the number of miles driven by a family based on the number of hours they traveled.

<table>
<thead>
<tr>
<th>Hours Traveled</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miles Driven</td>
<td>50</td>
<td>100</td>
<td>150</td>
<td>200</td>
<td>300</td>
</tr>
</tbody>
</table>

The table above shows a proportional relationship because for every hour traveled the family drives 50 miles. The ratio of miles per hour shows a constant of 50 miles per hour.
Example: The table below shows the number of Girl Scout Cookies Kelly sold compared to the total money collected at her troop’s annual cookie sale.

<table>
<thead>
<tr>
<th>X # Boxes Sold</th>
<th>Y $ Collected</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
</tr>
</tbody>
</table>

The graph shows a proportional relationship because:

1. The relationship forms a straight line when graphed.
2. The line passes through the origin.

Writing Equations to Represent a Proportional Relationship

If a proportional relationship is described by the set of ordered pairs that satisfies the equation $y = kx$, where $k$ is a positive constant, then $k$ is called the constant of proportionality.

Example:

You drive at a rate of 40 miles per hour. Write an equation that expresses the situation above. Let $y$ be the distance you drive. Let $x$ be the hours you drive.

How many miles you drive depends on the hours you drive.

Equation $\longrightarrow y = 40x$
Scale Drawings

**Scale Drawing:** A drawing in which all lengths between points or figures in the drawing are reduced or enlarged proportional to the lengths in the actual picture. A constant of proportionality exists between corresponding lengths of the two images.

**Reduction:** The lengths in the scale drawing are smaller than those in the actual object or picture.

**Enlargement/Magnification:** The lengths in the scale drawing are larger than those in the actual object or picture.

**One-to-one Correspondence:** Each point in one figure corresponds to one and only one point in the second figure.

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**Example:** Over the break, your uncle and aunt ask you to help them cement the foundation of their newly purchased land and give you a top-view blueprint of the area and proposed layout. A small legend on the corner states that 4 inches of the length corresponds to an actual length of 52 feet.

If the dimensions of the foundation on the blueprint are 11 inches by 13 inches, what are the actual dimensions in feet?

\[
\text{Width} \rightarrow \frac{\text{scale (in)}}{\text{actual (ft)}} = \frac{11}{x} = \frac{4}{4} = \frac{4x=57}{2} \quad x = 143 \text{ ft}
\]
Ratios & Proportional Relationships

Length → \( \frac{\text{scale (in)}}{\text{actual (ft)}} = \frac{4}{52} = \frac{13}{x} \)

\( \frac{4x}{4} = \frac{676}{4} \)

\( x = 169 \text{ ft} \)

*The actual width is 143 ft and the actual length is 169 ft.*

Finding the Scale Factor

The **scale factor** can be calculated from the ratio of any length in the scale drawing to its corresponding length in the actual picture. The scale factor corresponds to the unit rate and the constant of proportionality.

**Scale factor** - a scale written as a ratio without units in simplest form.

Scaling by factors *greater than 1* enlarge the segment and scaling by factors *less than 1* reduce the segment.

**Example:** Find the scale factor of a model sailboat if the scale is 1 inch = 6 feet.

1) **Write as a ratio of scale to actual.**

\( \frac{\text{scale (in)}}{\text{actual (ft)}} = \frac{1}{6} \)

2) **Convert units to be the same unit of measure.**

\( \frac{1 \text{ in}}{6 \text{ ft}} \rightarrow \frac{1 \text{ in}}{72 \text{ in}} \)

3) **Simplify if needed into a unit fraction.**

scale factor = \( \frac{1}{72} \)